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The speed of revolution of the line of apsides in the relative orbit of a close pair of stars that are flattened and elongated by rotation and tidal distortion is significant in the study of the internal structure of the stars. Every effort should be made, therefore, to increase our meager body of observational material on apsidal motion. Until recently, Y Cygni, studied of late by Dugan, Waterfield, Redman, and Russell,[1] was the only system for which we had clear evidence of an advancing periastron; but new photographic work at Harvard on the eclipsing binary RU Monocerotis, in verification of the visual observations of Dubiago and Martinoff at Kasan,[2] show not only that this close double star moves in an orbit of abnormally high eccentricity, but that the line of apsides is slowly revolving.

If t_1 and t_2 are the times of primary and secondary minima, P the orbital period, and e the eccentricity of the orbit, the longitude of periastron, ω, measured from the ascending node, is given by

$$\cos \omega = \frac{\pi[2(t_2-t_1)-P]}{2Pe},$$

to a sufficient approximation for binaries whose orbits are so little inclined to the line of sight that deep eclipses occur. The period of revolution of periastron is measured by the periodicity of the difference $d = t_2-t_1$, that is, by the cyclic shift of the secondary minimum with respect to the primary.

The accurate determination of $e \sin \omega$, and the consequent evaluation of e and ω separately, from the width and shape of the light curve at the two minima, is a difficult matter for which observations rarely suffice; and certainly the variations in ω cannot be measured satisfactorily from the character of the light curves during eclipses. The apsidal motion can best be determined from the relatively simple study of the times of eclipse for those systems whose light curves show two distinct minima in each revolution; but the determination of the dimensions and ellipticities of the component stars from analyses of

[1] For bibliography see Russell and Dugan, M. N., **91,** 212, 1930.

[2] A. N., **235,** 225, 1929.

the light curves also becomes necessary before the apsidal motion can be used to estimate the distribution of matter inside the stars.[3] For RU Monocerotis the data are as yet insufficient either to yield complete orbital elements, or to determine the period of revolution of the line of apsides. All that we can do at present is to show that the highly elongated orbit is revolving and to suggest that close and prolonged observation of this object is important in an attractive problem of celestial mechanics as well as in the astrophysics of stellar interiors.

RU Monocerotis is a tenth magnitude eclipsing variable in the position $6^h\ 50^m\ 56^s$, $-7°\ 25'.6$ (1932), spectral class A0, and period 3.58 days. It was discovered as an Algol variable many years ago by L. Ceraski. It has been observed by Blažko, Lehnert, Graff, Wendell, and Dubiago and Martinoff.[4] Wendell's early and rather fragmentary observations at Harvard show the astonishing and quite unparalleled situation that

$$d = t_2 - t_1 = 2.9(t_1 - t_2),$$

but it remained for Dubiago and Martinoff to call attention to the abnormality after finding a similarly high eccentricity indicated by their own visual observations. For nearly all other eclipsing binaries the ratio of intervals lies between 0.8 and 1.2.

Since $$e \geqq \frac{\pi}{2P}\left|\left(d - \frac{P}{2}\right)\right|,$$

the large displacement of the secondary minimum from the midpoint between successive primaries means that the orbit of RU Monocerotis has an eccentricity not less than 0.375, which is three times that of the orbit of the average binary of short period, and two and a half times that of the orbit of Y Cygni—perhaps the most eccentric among known eclipsing stars.

The observations on the Harvard plates, a total of 543 in number, extend through the interval from 1898 to 1931. Two estimates of magnitude were made on all plates during maximum light; four estimates on all plates during minima. The comparison stars are as follows:

[3] Russell, M. N., **88,** 641, 1928.

[4] For bibliography see A. N., **235,** 225, 1929; cf. also Nizhny-Novgorod, Veränd. St., **1,** No. 8–9, 1929.

	B.D.	Spectrum	Magnitude
a	−7°1616	A3	10.39
b	−7°1611	A0	10.92
c	. . .	A	11.75

The magnitudes of the comparison stars were estimated with reference to the astrographic sequence[5] at 7^h, $-2°.5$, on two plates of the AC series, one of the AM series, and one of the RH series. The magnitudes of the variable were derived almost wholly from plates of AC and AM series.

The observations on RU Monocerotis are given in the table where magnitudes for individual plates are listed for the minima, but for the maximum light the observations are grouped in means of four or five. Four successive intervals of time are treated separately in the table and in the accompanying figures which are based upon the tabulated results.

INTERVAL 1

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
0.050	10.55	5	2.176	10.59	5
0.232	10.57	5	2.275	10.55	4
0.506	10.56	5	2.358	10.55	4
0.816	10.54	5	2.436	10.60	1
1.022	10.54	5	2.503	11.17	1
1.092	10.55	5	2.523	11.25	1
1.206	10.55	5	2.569	11.21	1
1.278	10.55	4	2.585	11.25	1
1.450	10.56	4	2.610	10.81	1
1.624	11.17	1	2.702	10.58	5
1.659	11.25	1	2.774	10.54	5
1.676	11.17	1	2.852	10.53	4
1.740	10.66	1	2.962	10.57	4
1.766	11.09	1	3.102	10.60	4
1.782	10.53	5	3.372	10.57	5
1.862	10.57	5			
2.008	10.57	5			

[5] Harvard Annals, **85**, 145, 1924.

INTERVAL 2

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
0.150	10.54	5	2.390	10.52	4
0.398	10.56	5	2.478	10.68	1
0.674	10.55	5	2.575	10.80	1
0.948	10.56	5	2.598	10.89	1
1.308	10.54	5	2.612	10.87	1
1.420	10.74	1	2.652	10.66	1
1.487	10.68	1	2.668	10.54	4
1.524	10.76	1	2.820	10.54	4
1.529	10.81	1	2.970	10.53	4
1.543	11.09	1	3.048	10.60	4
1.547	10.89	1	3.158	10.58	4
1.579	11.00	1	3.514	10.59	5
1.617	11.21	1			
1.696	10.83	1			
1.711	10.71	1			
1.796	10.59	5			
1.956	10.57	5			
2.100	10.54	5			
2.272	10.58	4			

INTERVAL 3

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
0.024	10.55	5	1.591	11.25	1
0.102	10.57	5	1.596	11.17	1
0.198	10.58	5	1.623	11.04	1
0.280	10.53	5	1.666	10.75	1
0.382	10.55	5	1.677	10.87	1
0.432	10.56	5	1.701	10.72	1
0.626	10.56	5	1.770	10.55	5
0.754	10.53	5	1.880	10.57	5
0.790	10.55	5	1.988	10.56	5
0.914	10.54	5	2.064	10.55	5
1.038	10.52	5	2.116	10.56	5
1.178	10.56	4	2.168	10.54	5
1.275	10.52	4	2.270	10.60	4
1.370	10.57	4	2.325	10.60	4
1.424	10.50	1	2.370	10.58	4
1.495	10.87	1	2.422	10.58	1
1.509	10.89	1	2.464	10.81	1

INTERVAL 3—*Continued*

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
1.520	11.09	1	2.482	11.00	1
1.567	11.21	1	2.497	10.96	1
2.560	10.92	1			
2.569	11.25	1			
2.605	10.81	1			
2.696	10.54	5			
2.784	10.58	5			
2.898	10.54	5			
2.972	10.58	5			
3.100	10.59	5			
3.206	10.58	5			
3.278	10.54	5			
3.342	10.57	5			
3.478	10.60	4			
3.555	10.59	4			

INTERVAL 4

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
0.036	10.56	5	1.463	10.78	1
0.120	10.64	5	1.472	10.84	1
0.178	10.60	5	1.475	10.60	1
0.288	10.58	5	1.504	11.11	1
0.354	10.63	5	1.528	11.17	1
0.452	10.58	5	1.530	11.21	1
0.542	10.63	5	1.539	11.17	1
0.726	10.56	5	1.558	10.92	1
0.850	10.56	5	1.565	11.13	1
1.016	10.53	5	1.587	11.09	1
1.160	10.53	5	1.604	10.76	1
1.245	10.54	4	1.648	10.60	1
1.300	10.61	4	1.660	10.66	1
1.375	10.56	4	1.734	10.57	5
1.401	10.74	1	1.814	10.57	5
1.416	10.55	1	1.902	10.61	5
1.417	10.77	1	1.996	10.58	5
1.450	10.84	1	2.088	10.58	5
2.190	10.58	4	2.644	10.66	1
2.268	10.54	4	2.818	10.55	5
2.358	10.60	4	2.960	10.57	5

INTERVAL 4—*Continued*

Phase	Magnitude	Number of observations	Phase	Magnitude	Number of observations
2.412	10.62	4	3.108	10.54	5
2.462	10.58	1	3.310	10.57	5
2.473	10.68	1	3.358	10.64	4
2.496	10.74	1	3.448	10.58	4
2.497	10.76	1	3.530	10.57	4
2.501	10.87	1			
2.537	11.25	1			
2.547	11.17	1			
2.553	11.17	1			
2.554	11.34	1			
2.571	11.17	1			
2.596	11.11	1			
2.609	10.59	1			
2.632	10.83	1			

Phases were computed with the elements

$$\text{Minimum} = \text{J.D. } 2416016.845 + 3^{d}.5847\ \text{E}$$

The duration of minimum is short compared with the length of period (the radii of the stars, an approximate solution shows, are scarcely an eighth of the average distance between centers); accordingly, though maximum light is very satisfactorily measured for all four intervals, too few observations are available to define the light curve during eclipses, and hence the width of the minima cannot be estimated with sufficient precision to give a value of $e \sin \omega$. The times of the minima are not very accurately determined, but the following results are derivable from the four light curves:

Median date	$t_2 - t_1$	$\frac{t_2 - t_1}{t_1 - t_2}$
1902.0	2.69	3.02
1907.0	2.65:	2.85:
1914.0	2.60	2.65
1925.1	2.56	2.51

They indicate clearly the advance of periastron. If we take $e = 0.4$, then

$$\cos \omega = 0.547\ (2d - 3.585)$$

and for the foregoing median dates we have

$$\omega = 0°.9,\ 2°.8,\ 6°.4,\ 9°.0$$

If we take $e = 0.5$, then

$$\omega = 12°.2, 14°.4, 16°.8, 19°.0$$

The arc covered by the advancing periastron is too small and the observations too uncertain to make possible a good estimate of the period of apsidal revolution. It may be of the order of a thousand years.

With the Harvard polarizing photometer Wendell found the visual range of RU Monocerotis at primary eclipse to be

$$11^m.20 - 10^m.48 = 0^m.72$$

and the amplitude at the secondary perhaps a little less. Dubiago and Martinoff from observations made by the Argelander method find the two minima practically equal with a range of $0^m.60$. The mean photographic range derived from the Harvard observations is, within the uncertainties, the same for both eclipses:

$$11^m.24 - 10^m.56 = 0^m.68$$

The differences between the Harvard visual and photographic magnitudes at maximum and minimum are in agreement with the spectral class, A0. The amplitudes at the two eclipses show that we observe the nearly central eclipse of bodies essentially equal in size, luminosity, mass, and surface brightness. These similarities will simplify the discussion of the central concentration of matter within the stars when further observations have given us the period of apsidal revolution and accurate values of the radii of the component stars.

For the four groups of photographic observations the mean values of maximum light are, respectively:

Median date	Photographic magnitude	Mean error	Number of observations
1902.0	10.557	0.004	98
1907.0	10.559	0.006	73
1914.0	10.560	0.004	157
1925.1	10.578	0.006	140
All	10.565	0.003	468

Both the maximum and minimum are fainter by approximately two hundredths of a magnitude during the fourth interval. The difference probably reflects merely the introduction of series of plates with color equations different from the average for the earlier intervals. About sixty plates of the series AX, AY, RB, and RH were used after 1920.

Of ellipticity or reflection effect there is no evidence, which is in keeping with our past experience with stars that are small relative to their separation. At periastron, however, the bodies for several hours are separated by less than two diameters; and yet during that interval of time, there appears to be no visible effect on the total light of the system, for we compute from the data in the foregoing table (134 observations) that the maximum light is then essentially constant at 10.563. The absence of a periastron brightening, such as is frequently observed for binaries with much lower eccentricities, can be taken, in connection with the very slow revolution at periastron, to indicate that the components of RU Monocerotis have high central concentration.

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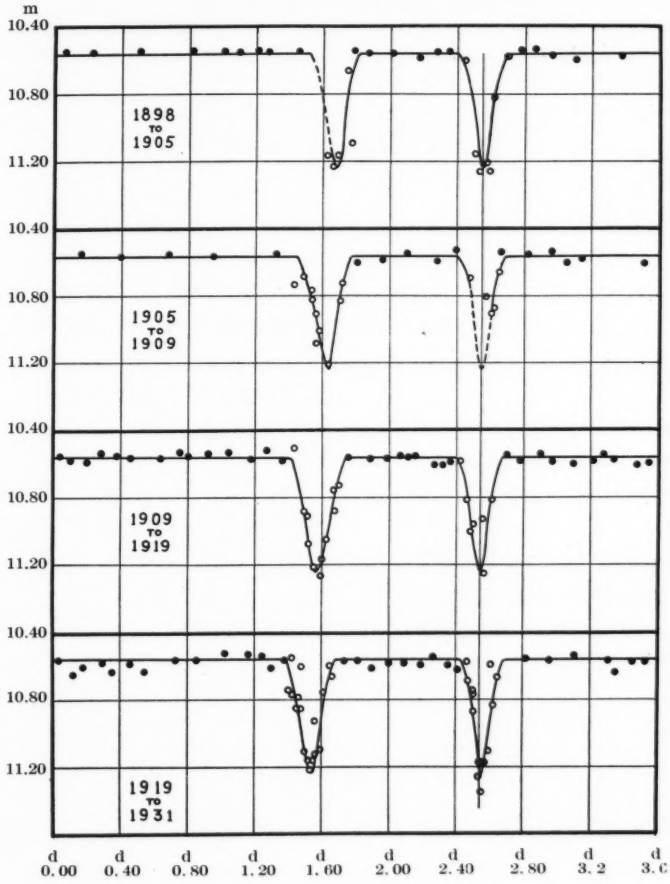
Photographic light curve of RU Monocerotis, illustrating the anomalous separation of the minima and its change with time. Ordinates are photographic magnitudes; abscissae are days. Open circles in the minima represent single observations; the dots represent groups of four and five observations.